Explaining exhaustivity in terms of Attentional Quantity

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The standard pragmatic recipe for exhaustivity implications is based on Grice's maxim of Quantity plus a competence assumption. In recent years a number of serious problems for this recipe have been (re)discovered. We present a formal, unifying solution that, moreover, generates existing exhaustivity operators from more basic, pragmatic assumptions.

1. Five problems for the standard recipe

- **Granularity:** Sentences that are classically (informationally) equivalent may nevertheless differ in exhaustivity implications (Van Rooij & Schulz, 2006; Katzir & Singh, 2013):

  (1) Q: Who (of John, Mary and Bill) was at the party?
  - a. A: John. *(Exh.: Mary and Bill weren't.)*
  - b. A: John, or both John and Mary. *(Exh.: Bill wasn't.)*
  - c. A: John, or everyone. *(Exh.: if Mary or Bill, then everyone.)*

- **Questions:** Questions arguably lack an informational intent for Quantity to apply to, but they nevertheless imply exhaustivity (e.g., Biezma & Rawlins, 2012):

  (2) Was John there, or Mary? *(Exh.: not both)*

  (Note that what is *implied* is not necessarily *implicated*, and may well be *presupposed*.)

- **Hints:** Hints are arguably exempt from Quantity, but do imply exhaustivity (Fox, 2014):

  (3) There is money in box 20 or 25. *(Exh.: not both)*

  See Fox for a rebuttal of a number of possible coping strategies for the standard recipe.

- **Competence:** Exhaustivity occurs without a competence assumption (Westera, 2013b):

  (4) Q: You may not know this, but who (of John, Mary and Bill) was at the party?
  A: John and Mary were there. *(Exh.: Bill wasn't)*

  See Westera for an argument that purported evidence in favour of the reliance of exhaustivity on a competence assumption (e.g., Breheny et al. 2013) has been misinterpreted. (We set aside the important role of *intonation*; we favour an account according to which a final fall conveys compliance with the maxims, see Westera, 2013a.)

- **Symmetry:** Exhaustivity implications occur on responses to symmetrical questions:

  (5) Q: Of John, Mary and Bill, who was and who wasn't at the party?
  A: John and Mary were there. *(Exh.: Bill wasn't)*

  The initiative suggests that negative answers are relevant too, but if so, then compliance of the answer with Quantity would imply A's inability to give a negative answer, preventing exhaustivity. (We assume, like Horn (1989), a.o., but unlike Chierchia et al. (2012), that relevance is not *always* symmetrical; but in (5), given the initiative, it presumably is.)

A sixth problem, to which we will *not* propose a (general) solution, is that of *embedded* exhaustivity. We consider a globalist solution to the above problems worth pursuing regardless of whether embedded and unembedded cases constitute a single phenomenon.

2. A new recipe based on attention We adopt the common assumption that a disjunction evokes its disjuncts as *alternatives* (e.g., Aloni, 2007), of which we conceive as possibilities to which a speaker intends to *draw attention* (Ciardelli et al. 2009). As a communicative intention this must be governed by an appropriate set of maxims. We define these in Montague's *Intensional Logic* (IL), with doxastic speaker modalities ($\Box$, $\Diamond$) and set-theoretical shorthands. Let a, b,... be constants/variables of type $\langle s,t\rangle$ (propositions) and $\mathcal{A},\mathcal{B}$,... of type $\langle\langle s,t\rangle,t\rangle$. To illustrate, we first define several of Grice's maxims, or *I(nformation)-maxims*. Given an informational intent p and a question under discussion (QUD) $\mathcal{Q}$:

I-Quality$(p) = \Box^{\vee}p$ I-Relation$(\mathcal{Q}, p) = \mathcal{Q}(p)$
I-Quantity$(\mathcal{Q}, p) = \forall q\,\big((\text{I-Quality}(q) \wedge \text{I-Relation}(\mathcal{Q}, q)\big) \rightarrow (p \subseteq q)\big)$

In words: intend to share only information that you believe is true (I-Quality) and that answers the QUD (I-Relation); and share *all* such information (I-Quantity). (We ignore *indirect* answers, which would require a more sophisticated I-Relation and I-Quantity.)

For an *attentional* intent $\mathcal{A}$ (evoked alternatives) we define a similar set of maxims:

A-Quality$(\mathcal{A}) = \forall a\,(\mathcal{A}(a) \rightarrow \Diamond({}^{\vee}a \wedge \forall b\,((\mathcal{T}(b) \wedge b \subset a) \rightarrow \neg^{\vee}b)))$
A-Relation$(\mathcal{Q}, \mathcal{A}) = \forall a(\mathcal{A}(a) \rightarrow \mathcal{Q}(a))$
A-Quantity$(\mathcal{Q}, \mathcal{A}) = \forall a((\text{A-Quality}(\{a\}) \wedge \text{A-Relation}(\mathcal{Q}, \{a\})) \rightarrow \mathcal{A}(a))$

In words: intend to draw attention only to answers to the QUD (A-Relation) that you consider possible independently of any stronger possible answers (A-Quality); and draw attention to *all* of those (A-Quantity). The independence requirement in A-Quality can be regarded as a weakened version of *Hurford's Constraint* (HC), according to which no disjunct may entail the other. Whereas HC is prima facie falsified by examples like (1b,c), the maxim of A-Quality may permit such cases, provided the weaker disjunct is considered independently possible, a requirement we intend to reflect considerations of attentional economy. (For the strangeness of cases like "a lion or a mammal", which motivated HC, we favour an account based on levels of categorization; see Rosch, 1978.)

Let an *admissible model* be a model for IL in which the foregoing definitions (grey boxes) are valid (cf. meaning postulates for Montague), the denotations of $\mathcal{Q}$ are closed under (infinitary) intersection, and the set of worlds is sufficiently large to distinguish all contingent first-order formulae (needed for Fact 2). We can prove that compliance with A-Quantity means that every answer to the QUD to which no attention is drawn, is believed to be either false or only ever true together with some stronger answer to which attention is drawn:

Fact 1. In all admissible models **M**, for arbitrary constants $\mathcal{A}, \mathcal{Q}$:
$\mathbf{M} \models \text{A-Quantity}(\mathcal{A}, \mathcal{Q}) \rightarrow \forall a\,((\mathcal{Q}(a) \wedge \neg\mathcal{A}(a)) \rightarrow \Box(\neg^{\vee}a \vee \exists b(\mathcal{A}(b) \wedge (b \subset a) \wedge {}^{\vee}b)))$

3. Solutions to the five problems

- **Granularity**: For (1a,b,c), let $\mathcal{Q}$ denote the closure of $\{{}^{\wedge}Pj, {}^{\wedge}Pm, {}^{\wedge}Pb\}$ under $\cap$ and $\cup$ (here Pj translates "John was at the party", etc.). Let $\mathcal{A}$ denote the sets $\{{}^{\wedge}Pj\}$ for (1a), $\{{}^{\wedge}Pj, {}^{\wedge}(Pj \wedge Pm)\}$ for (1b) and $\{{}^{\wedge}Pj, {}^{\wedge}(Pj \wedge Pm \wedge Pb)\}$ for (1c). The predictions are as desired; for reasons of space we state the formal result only for (1c):

  Fact 2. For all admissible models **M** for (1c), with $\mathcal{A}$ and $\mathcal{Q}$ as just explained:
  $\mathbf{M} \models \text{A-Quantity}(\mathcal{A}, \mathcal{Q}) \rightarrow (\neg Pm \vee (Pj \wedge Pm \wedge Pb)) \wedge (\neg Pb \vee (Pj \wedge Pm \wedge Pb))$
- **Questions:** For (2), take $\mathcal{Q} = \{{}^{\wedge}Pj, {}^{\wedge}Pm, {}^{\wedge}(Pj \wedge Pm)\}$ and $\mathcal{A} = \{{}^{\wedge}Pj, {}^{\wedge}Pm\}$. Although I-Quantity does not apply to questions, A-Quantity does, and delivers the right result.
- **Hints:** This we solve only conceptually: we propose that (3) is exempt from I-Quantity but not A-Quantity: while withholding information is part of their job, a quizmaster who doesn't draw attention to all live options is guilty of misleading (and should be fired).
- **Competence:** Exhaustivity was derived in Facts 1 & 2 without a competence assumption. The standard recipe would rely on this assumption to obtain $\Box\neg$ from $\neg\Box$, where the latter would be obtained from I-Quantity. But whereas the antecedent of I-Quantity contains a $\Box$ (from I-Quality), the negation of which is merely $\neg\Box$, the antecedent of A-Quantity contains a $\Diamond$, the negation of which is $\neg\Diamond$, equivalent to $\Box\neg$.

- **Symmetry:** For (5A), let $\mathcal{A} = \{{}^\wedge(Pj \wedge Pm)\}$. Suppose that $\mathcal{Q}$ is symmetrical, i.e., $\{{}^\wedge Pj, {}^\wedge\neg Pj, {}^\wedge Pm, {}^\wedge\neg Pm, {}^\wedge Pb, {}^\wedge\neg Pb, \ldots\}$ (suitably closed). Then we can prove:

  > **Fact 3.** For all admissible models **M** for (5A), with $\mathcal{A}$ and $\mathcal{Q}$ as just explained:
  > $$\mathbf{M} \models \neg\,\text{A-Quantity}(\mathcal{A}, \mathcal{Q})$$

  Since (5A) cannot compliantly address the symmetrical QUD of (5Q), it must, contrary to our supposition, be addressing some *accommodated* QUD. A reasonable assumption is that the answerer chose to split the original QUD into two: *who was present* (like $\mathcal{Q}$ in (1)) and *who was absent*. This is a valid *strategy* (Roberts, 1996), and it favours brevity by letting negative answers be conveyed as an exhaustivity implicature – in a sense the symmetry problem solves itself. Lest this seem trivial: unlike the A-maxims, the I-maxims fail to inform us (or an addressee) that a different QUD is accommodated: the informational intent $p = {}^\wedge(Pj \wedge Pm)$ can comply with the I-maxims relative to the symmetrical $\mathcal{Q}$.

4. Formal comparison For easy comparison we capture the consequent of fact 1, strengthened by factivity (that the beliefs are true), in an 'exhaustivity operator':

$$\text{EXH}(\mathcal{Q}, \mathcal{A}) \stackrel{\text{def}}{=} {}^\wedge\forall a \begin{pmatrix} (\mathcal{Q}(a) \wedge \neg\mathcal{A}(a)) \rightarrow \\ (\neg{}^\vee a \vee \exists b(\mathcal{A}(b) \wedge (b \subset a) \wedge {}^\vee b)) \end{pmatrix} \quad \text{(equivalent to } \bigcap_{\substack{a \in \mathcal{Q} \\ a \notin \mathcal{A}}} (\overline{a} \cup \bigcup_{\substack{b \in \mathcal{A} \\ b \subset a}} b))$$

We intend this merely as a notational shorthand, not as a grammatical operator. Now:

> **Fact 4.** For all admissible **M**, if p and $\mathcal{A}$ comply with the I/A-maxims relative to $\mathcal{Q}$:
> a. if $\mathcal{A}$ denotes the singleton set $\{p\}$:
> $\mathbf{M} \models p \cap \text{EXH}(\mathcal{Q}, \mathcal{A}) = \text{EXH}_{\text{mw}}(\mathcal{Q}, p)$ (EXH$_{\text{mw}}$ as discussed in Spector, 2016)
> b. if for some predicate P and domain D, $\mathcal{Q} = \{{}^\wedge Px \mid x \in D\}$ closed under $\cap$, and a context-change potential C assigns all and only $a \in \mathcal{A}$ to a discourse referent:
> $\mathbf{M} \models p \cap \text{EXH}(\mathcal{Q}, \mathcal{A}) = \text{EXH}_{\text{R\&S}}(P, C)$ (EXH$_{\text{R\&S}}$ from Van Rooij & Schulz, 2006)

That is: (a) with information only, our EXH is conservative; and (b) the A-maxims generate/explain the more advanced operator of Van Rooij and Schulz, which they merely stipulate – they explain only EXH$_{\text{mw}}$, in terms of the standard recipe. Note that if p or $\mathcal{A}$ violates a maxim relative to $\mathcal{Q}$, our operator may deliver nonsense, and others remain unexplained.

5. References • Aloni, M. (2002). Free choice in modal contexts. *SuB7.* • Biezma, M., & Rawlins, K. (2012). Responding to alternative and polar questions. *Ling. & Phil.* 35. • Breheny, R., Ferguson, H., & Katsos, N. (2013). Taking the epistemic step: [...] *Cognition* 126. • Ciardelli, I., Groenendijk, J., & Roelofsen, F. (2009). Attention! *Might* in inquisitive [...]. *SALT XIX.* • Chierchia, G., Fox, D., & Spector, B. (2012). The grammatical view [...]. In Maienborn et al. (Eds.), *Semantics: An international handbook* [...] vol. 2. • Fox, D. (2014). Cancelling the Maxim of Quantity: [...]. *Sem. & Prag.* • Horn, L.R. (1989). *A natural history of* [...]. Uni. of Chicago. • Katzir, R. & Singh, R. (2013). Hurford Disjunctions: [...]. *SuB18.* • Roberts, C. (1996). Information structure [...]. *OSU w.p. in ling.* 49. • Rosch, E. (1978). Principles of categorization. In Rosch & Lloyd (Eds.) *Cognition and categorization.* Lawrence Erbaum. • Spector, B. (2016). Comparing exhaustivity operators. *Semantics and Pragmatics* 8. • Van Rooij, R., & Schulz, K. (2006). Pragmatic meaning and non-monotonic [...]. *Linguistics and Philosophy* 29(2). • Westera, M. (2013a). 'Attention, I'm violating a maxim!' [...]. SemDial 17. • Westera, M. (2013b). Where the air is thin, [...]. In Aloni et al. (Eds.) *The dynamic, inquisitive* [...].