A surface-scope analysis of authoritative readings of modified numerals

Brian Buccola and Andreas Haida
Hebrew University of Jerusalem

Overview. This paper concerns a well-known puzzle regarding the interpretation of superlative modified numerals (e.g. *at most n*) in the scope of existential modals (e.g. *allow*). We present new data that broaden the scope of the puzzle, and we offer a new (but partial), surface-scope solution based on a conservative amendment of the recursive exhaustification approach to free-choice disjunction laid out in Fox 2007.

Basic puzzle. A well-known puzzle regarding the interpretation of *at most n* is that when it occurs in the scope of an existential deontic modal, as in (1a), the resulting sentence can have an *authoritative reading*, characterized by two kinds of inferences: (i) an upper-bound (UB) inference, viz. that you're not allowed to draw 4 or more cards, and (ii) a free-choice (FC) inference, viz. that you're allowed to draw any number of cards in the range [0, 3] (Geurts and Nouwen 2007; Büring 2008; Nouwen 2010; Penka 2014; Kennedy 2015). On standard assumptions about the meanings of *allow* and *at most 3*, however, a surface-scope analysis of (1a) predicts only a weak literal meaning: there is some permissible world in which you draw 3 cards or fewer (notated henceforth as $\Diamond[\leq 3]$); neither UB nor FC logically follow from this. ((1a) also has an *ignorance reading*, which we ignore right now; this is the reading that would result from inverse scope of *at most 3* above *allowed*; Penka 2014; Kennedy 2015.)

(1) a. You're allowed to draw at most 3 cards.
b. #You're allowed to draw at least 5 cards.

Curiously, *at least n* does *not* give rise to an analogous authoritative reading: (1b) cannot be used to convey that you're allowed to draw 5 or more cards, with FC in the range [5, …], and a lower bound (LB) that prohibits drawing 4 cards or fewer. Thus, in contexts that do not support an ignorance reading, (1b) is infelicitous (hence the '#').

The puzzle, then, has two parts: (i) why does (1a) have an upper-bounded authoritative reading, and (ii) why does (1b) *not* have an analogous lower-bounded authoritative reading?

A previous analysis. Penka (2014), quite reasonably, takes *at most* to be the 'oddity' in this puzzle and solves it by decomposing *at most* into a negative component, *ANT*, plus *at least*, so that (1a) is analyzed with split scope as *ANT 3 [… allow [… at least …]]*, which means 'you're not allowed to draw more than 3 cards' ($\neg\Diamond[\geq 4]$), thus entailing an UB. The FC inferences follow from neo-Gricean reasoning, given certain assumptions about the scales responsible for generating alternatives. That (1b) has no authoritative reading, in particular no LB, follows because *at least* is assumed to have no analogous negative component: *allow [… at least 3 …]* receives only a weak interpretation ($\Diamond[\geq 3]$).

New observations. The novel data in (2) show that a variety of expressions, beyond *at most n*, can give rise to authoritative readings. For instance, (2a) shows that two 'antonyms' can each have authoritative readings with opposite bound inferences, viz. that you may not arrive {earlier/later} than 8:00 PM (in addition to their FC inferences, viz. that you may arrive at 8:00 PM and you may arrive {later/earlier}). (2b) exhibits double-boundedness, as well as FC effects within the allowable range, viz. that you can borrow any amount between \$1,000 and \$5,000, but you may not borrow less than \$1,000 or more than \$5,000. Finally, (2c) shows that, with the definite article, the lower-bound superlative modifier (*at … least*) *can* indeed convey a LB authoritative reading.

(2) a. You may arrive {at the earliest/at the latest} at 8:00 PM.
b. You're allowed to borrow between \$1,000 and \$5,000 from this bank.
c. You can have at the least 3 children (and still qualify for the tax exemption).

Penka's approach cannot capture these authoritative readings without *ad hoc* assumptions, e.g. that *at the earliest* decomposes into 'not earlier than', while *at the latest* decomposes into 'not later than'; that *between m and n* decomposes into 'not fewer than m and not more than n'; and that *at the least* (but not *at least*) decomposes into 'not fewer than'.

Note also that an inverse-scope analysis would predict ignorance inferences across the board. This is because when expressions like *at most n*, *at the latest*, and so on take sentential (or widest) scope, ignorance readings obligatorily emerge, as in *Mary drew at most 3 cards*, *Bill arrived at the latest at 8:00 PM*, and so on. While ignorance readings are certainly available for the examples in (2), they are distinct from the authoritative readings we want to derive.

The conclusion we draw is that it is not *at most* which is the oddity in this puzzle, nor is it *at least* (per se), but rather more specifically the modified numeral *at least n*. In other words, it seems advantageous to analyze all the examples above in a uniform way, by retaining a standard morphosyntax and semantics for expressions like *at most n* and locating the source of authoritative inferences elsewhere, rather than proposing a special morphosyntactic structure for all the many expressions that can give rise to authoritative readings.

Proposal. Our starting point is the observation that disjunction in the scope of an existential modal licenses similar FC and 'bound' inferences. For example, in a context where the relevant desserts are cake, gelato, and pie, (3) licenses the inferences (i) that you're not allowed to have pie ($\neg \Diamond p$), and (ii) that you're allowed to have cake, and you're allowed to have gelato ($\Diamond c \wedge \Diamond g$). Fox (2007) presents a surface-scope analysis of free-choice disjunction in which the literal meaning of (3), $\Diamond[c \vee g]$, is strengthened via recursive exhaustification: (3) is parsed as *exh [exh [... allow [... cake or gelato]]]*. Though Fox (2007) does not discuss inference (i), it is clear that, if *You're allowed to have pie* is an alternative to (3) (as it would be on a theory of alternatives like Katzir's (2007)), then the first (inner) round of *exh* correctly derives inference (i). The second (outer, recursive) round of *exh* derives inference (ii), as Fox (2007) shows. (We ignore the inference $\neg \Diamond (c \wedge g)$, which also follows from the first round of *exh*.)

(3) You're allowed to have cake or gelato.

Given the inferential (and syntactic) similarity between (1a) (henceforth *S*) and (3), we propose a surface-scope analysis of *S*, in which the literal meaning $\Diamond[\leq 3]$ is strengthened by recursive exhaustification into $[\![\textit{exh [exh S]}]\!] = \Diamond[\leq 3] \wedge \neg \Diamond [\geq 4] \wedge \Diamond[= 3] \wedge \Diamond[= 2] \wedge \cdots$.

For the first (inner) round of exhaustification, we assume that the set of alternatives to *S* (alt(S)) includes those obtained by replacing *at most* with *exactly* and *3* with any numeral, i.e. $\{\Diamond[= n] \mid n \in \mathbb{N}\} \subseteq \text{alt}(S)$ (cf. Schwarz 2016). All innocently excludable (IE) alternatives in alt(S) are excluded in the standard way (Fox 2007), which derives an UB, because $\Diamond[= 4]$, $\Diamond[= 5]$, etc. are all IE. (This part is analogous to excluding *pie* above.)

For the second (recursive/outer) round of exhaustification, if we assumed, as Fox (2007) does, that alt(*exh S*) is the set of all strengthened alternatives to *S*, i.e. $\{\text{exh}(\text{alt}(S))(p) \mid p \in \text{alt}(S)\}$, then it turns out that we would not derive strong enough FC inferences. Here's why. On that assumption, alt(*exh S*) would include the set of all exhaustified *exactly n* alternatives, i.e. $\{\text{exh}(\text{alt}(S))(\Diamond[= n]) \mid n \in \mathbb{N}\}$. The exhaustified meaning of $\Diamond[= n]$ is $\Diamond[= n] \wedge \neg \Diamond [< n] \wedge \neg \Diamond [> n]$. The set of all such propositions is therefore $\{\Diamond[= n] \wedge \neg \Diamond[< n] \wedge \neg \Diamond[> n] \mid n \in \mathbb{N}\}$. Everything in this set is IE, so we exclude everything, yielding for the strengthened meaning of *S* the proposition $\Diamond[\leq 3] \wedge \neg \Diamond [\geq 4] \wedge \bigwedge\{\neg(\Diamond[= n] \wedge \neg \Diamond [< n] \wedge \neg \Diamond [> n]) \mid n \in \mathbb{N}\}$. The last, big conjunct is equivalent to $\bigwedge\{\Diamond[= n] \rightarrow (\Diamond[< n] \vee \Diamond[> n]) \mid n \in \mathbb{N}\}$. The problem is that this 'free choice' condition is satisfied even in a scenario where, say, exactly 1 and exactly 3 are allowed, but exactly 2 is forbidden, or exactly 1 and exactly 2 are allowed, but exactly 3 is forbidden. More generally, the inference we derive is that there is free choice between (at least) two indeterminate numbers in the range [0, 3], but not every number in that range. This inference is far too weak to capture the intuitive FC inference of S.

Instead, we propose that the set of alternatives for the second exhaustification includes not simply all strengthened propositions taken from alt(S), but rather all strengthened propositions taken from the *disjunctive closure* of alt(S), i.e. $\text{alt}(exh\ S) = \{\text{exh}(\text{alt}(S))(p) \mid p \in \text{alt}(S)^{\vee}\}$. The effect of this amendment is to introduce *weaker* propositions into the alternative set, so that their exclusion results in stronger inferences overall. For example, $p = \diamond[=0] \vee \diamond[=1] \vee \diamond[=3]$ is in $\text{alt}(S)^{\vee}$. $\text{exh}(\text{alt}(S))(p)$, which is now in $\text{alt}(exh\ S)$, entails $\neg \diamond [=2]$; hence, negating $\text{exh}(\text{alt}(S))(p)$, together with the strengthened assertion $\text{exh}(\diamond[\leq 3]) = \diamond[\leq 3] \wedge \neg \diamond [\geq 4]$, entails $\diamond[=2]$. Thus, the overall meaning derived for S is $\diamond[\leq 3]$ (basic meaning), plus $\neg\diamond[\geq 4]$ (first *exh*: UB), plus $\diamond[=3] \wedge \diamond[=2] \wedge \diamond[=1] \wedge \diamond[=0]$ (second *exh*: FC).

To derive this effect, we can assume that the set of alternatives to any sentence S is the disjunctive closure of the set of alternatives derived from S by structural manipulation, e.g. along the lines of Sauerland 2004 or Katzir 2007. For non-recursive exhaustification, this assumption shows no effects because a disjunction of alternatives is IE iff each disjunct is IE.

Importantly, our amendment does not disrupt the analysis of FC disjunction like (3), because in this case the alternative set is already closed under disjunction. In addition, our proposal extends naturally to all the cases in (2). However, it also incorrectly predicts a LB authoritative reading of (1b). Thus, if our proposal is on the right track, then, together with our new empirical observations, it brings to light a new angle on an old puzzle: What prevents *at least n* from participating in authoritative readings under an existential modal?

Ignorance readings. Our proposal is particularly close to a proposal, cast in neo-Gricean terms, that Schwarz (2016) entertains, but ultimately discards, for deriving ignorance readings of *at least*. (We should note, however, that Schwarz does not analyze *at most*, nor authoritative readings of modified numerals.) The reason he discards the account is because it—and ours—predicts *total* ignorance for a sentence like *Mary drew at least/at most 3 cards* (just like our account predicts total FC in the embedded case). Schwarz offers several arguments in favor of deriving partial, not total, ignorance. However, we disagree with his conclusions and will present counterarguments showing that his data can be explained by pruning the alternative set on the basis of relevance.

References

Büring, Daniel (2008). "The Least *at least* Can Do". In: *Proceedings of the 26th West Coast Conference on Formal Linguistics*. Ed. by Charles B. Chang and Hannah J. Haynie. Somerville, MA: Cascadilla Proceedings Project, pp. 114–120. URL: http://www.lingref.com/cpp/wccfl/26/paper1662.pdf.

Fox, Danny (2007). "Free Choice and the Theory of Scalar Implicatures". In: *Presupposition and Implicature in Compositional Semantics*. Ed. by Uli Sauerland and Penka Stateva. Palgrave Studies in Pragmatics, Language and Cognition Series. Palgrave Macmillan. Chap. 4, pp. 71–120. DOI: 10.1057/9780230210752_4.

Geurts, Bart and Rick Nouwen (2007). "At least et al.: The semantics of scalar modifiers". In: *Language* 83, pp. 533–559. URL: http://www.jstor.org/stable/40070901.

Katzir, Roni (2007). "Structurally-defined Alternatives". In: *Linguistics and Philosophy* 30.6, pp. 669–690. DOI: 10.1007/s10988-008-9029-y.

Kennedy, Chris (2015). "A "de-Fregean" semantics (and neo-Gricean pragmatics) for modified and unmodified numerals". In: *Semantics and Pragmatics* 8.10, pp. 1–44. DOI: 10.3765/sp.8.10.

Nouwen, Rick (2010). "Two kinds of modified numerals". In: *Semantics and Pragmatics* 3.3, pp. 1–41. DOI: 10.3765/sp.3.3.

Penka, Doris (2014). "*At most* at last". In: *Proceedings of Sinn und Bedeutung 19*. Ed. by Eva Csipak and Hedde Zeijlstra, pp. 463–480. URL: http://nbn-resolving.de/urn:nbn:de:bsz:352-0-304168.

Sauerland, Uli (2004). "Scalar Implicatures in Complex Sentences". In: *Linguistics and Philosophy* 27.3, pp. 367–391. DOI: 10.1023/B:LING.0000023378.71748.db.

Schwarz, Bernhard (2016). "Consistency preservation in Quantity implicature: the case of *at least*". In: *Semantics and Pragmatics* 9.1, pp. 1–47. DOI: 10.3765/sp.9.1.